greater power than the Melbourne one, has been lately constructed, and is now daily expected to arrive from England; and as the difference of longitude between Melbourne and Sydney has been accurately measured by means of the telegraph, it will be easy to compare its longitude results with our own. At the Adelaide Observatory no special observations for longitude have as yet been taken. There, also, the Government is just about to order a transit circle, the telescope of which will be somewhat larger than our own; and as the difference of longitude has also been telegraphically determined, its results will be immediately comparable with our own. The acquisition of two such fine instruments by the neighbouring Observatories is a matter for congratulation, and will enable them in future to take their share of the immense work to be done in the Southern hemisphere, an undue proportion of which has lately fallen to Melbourne.

Art. IX.—*Notes on Iron Arches.*

By W. C. Kernot, M.A., C.E.

[Read 25th September, 1875.]

The application of iron, and especially of wrought iron, to bridge-building is deservedly ranked as one of the most notable of those innovations in civil engineering practice that have been made in modern times. It has enabled us to cross chasms of enormous width and depth, and to erect safe and commodious structures in situations and under circumstances which would in many cases totally preclude the employment of the materials known to the bridge-builders of an earlier date. So long as stone and brick were the only available materials, the engineer was confined in his choice to small spans, and to sites where a thoroughly sound foundation was easily attainable. The largest stone arch ever constructed, as far as I can ascertain, is considerably less than 250 feet span, while iron structures on the arch or girder principle of double, and on the suspension principle of three times, this span are by no means uncommon, and we are yet far from approaching the limit of the maximum possible span in this material. Moreover, iron bridges can be employed with perfectly satisfactory results in sites where, from lack of headway, defective foundation, or other local peculiarity, a stone or brick structure would be quite out of the question; and the selection of lines of communication is thus greatly facilitated, and their length and cost consequently diminished.

The most usual form in which iron is employed for bridge purposes is the beam or girder, consisting of two parallel flanges united by a vertical web, consisting either of a continuous plate or of a series of diagonal bars. The average cross-section of such a girder is shown in Fig. 1. In a girder supported at each end the upper flange is in compression, like a pillar; the lower flange is in tension, like a chain—indeed, in some girders the lower flange actually consists of a chain; while the web is in a somewhat complex state of stress, being compressed in an oblique direction, and extended in another oblique direction at right-angles to the first. In girders with parallel flanges, subject to distributed loads of the usual kind, the compression and tension of the flanges attain maximum values at the centre of the span, and diminish toward the ends, while the web stresses are but small at mid-span, and increase towards the supports. Hence the cross-sections of a theoretically perfect girder, at the centre and the end, would be of the forms represented by Figs. 2 and 3 respectively.

Occasionally girders are made of varying depth, as shown in Fig. 4, the bottom flange being retained straight, while the top one is curved; and if this curve be properly designed in view of the special distribution of load anticipated, the following results will be secured:—

1. The tension on the lower flange will be uniform throughout.

2. The compression on the upper flange will be nearly uniform throughout, increasing slightly towards the ends.

3. The stresses on the web will vanish, and the web may consequently be dispensed with.

We have now left but two flanges, one curved and the other straight, like a bow and its string, and these two flanges will together contain rather less metal than an ordinary parallel girder of equal depth and strength.

In the girder as thus modified, the compression of the upper or curved flange at the end of the girder may be resolved into two forces—one vertical, which is balanced by the upward reaction of the support, and one horizontal, which is antagonised by the tension of the lower flange.

Let us now suppose the lower flange to be removed, thus reducing the amount of material employed, in the case of wrought iron, by nearly one-half, and we shall find the upper or curved flange alone to be fully competent to endure the load, provided that the supports or abutments be so constructed as to resist the horizontal as well as the vertical resolved parts of the compression at the ends of the remaining flange.

We have now gradually transformed our structure from an ordinary parallel girder with two flanges and a web into an iron arch, and in so doing we have reduced the amount of material theoretically requisite by almost exactly one-half. From this it follows that as far as material is concerned an arch is a far more economical means of supporting an unvarying load than a girder whenever a good abutment is available capable of resisting a horizontal thrust as well as a vertical pressure.

In working this form of bridge out in practice we are, however, met by certain difficulties, in order to overcome which we are obliged to relinquish a part of the economic advantage which theory indicates.

1. The arch will be exposed to variations of temperature, which may amount to as much as 100° Fahrenheit in a Victorian climate, and which will cause considerable variations of dimension through alternate expansion and contraction of the metal. These changes of dimension, though perfectly harmless in the case of girders free to elongate horizontally, may lead to very serious if not dangerous results in the case of arches placed between immovable abutments; and it is imperatively necessary to take such precautions as shall prevent injury to the structure under extreme variations of temperature.

The most thorough method of meeting this requirement is to divide the arch rib into two parts at the crown, and connect these two parts together, and the ends of the arch to the abutment by joints possessing the character and performing the functions of hinges (see Fig. 5). The arch as thus modified will rise slightly when the temperature increases, and fall slightly when the temperature diminishes, and the change of temperature will be powerless to produce any sensible variation in the stress to which the material is subject.

Sometimes the arch rib is made with hinges at the ends only, and the elasticity or spring of the iron itself is depended upon in lieu of the central hinge, and by properly proportioning the transverse dimensions of the rib it is possible to ensure that within a given range of temperature the metal shall not be strained to any dangerous extent. An arch of this second kind will be manifestly less economical in material than one of the first, seeing that it is required to endure considerable stresses due to variations of temperature over and above those due to the load supported. Nevertheless there are certain practical considerations—such as simplicity of construction, facility of erection, &c.—which may be reasonably held in some cases to justify its use in preference to the more theoretically perfect form previously described.

2. A second difficulty arises when in addition to the unvarying or dead load, consisting of the weight of the structure itself, we desire the arch to support a varying, or as it is often termed a live, load, such as the weight of a crowd of people, a mob of cattle, or a railway train in motion. So long as the load is a perfectly unvarying one, no matter how irregularly it may be distributed, it is possible to adopt a form of arch which will be perfectly suited to the load to be carried, but with a varying load, occuping the same position and affecting the structure in the same way for no two successive instants, such adaptation is manifestly impossible. Hence the rib will be subjected to a cross-bending action, and be required to act to a considerable extent as a beam as well as to perform its proper functions as an arch; and this cross-bending action will be severe in small structures in which the live load is equal to or greater than the unvarying or dead load, but will become unimportant in gigantic works in which the live load becomes but an insignificant fraction of the total weight carried. Thus it will be seen that while in large structures we may reasonably expect to realise nearly the whole of the theoretical economic advantage of the arch over the girder, in small ones the additional metal necessary in order to provide for the extra stresses due to the varying distribution of the moving or live load will greatly diminish, if not altogether annul, the superior economy of the arch as compared with its competitor.

I may here parenthetically remark that there is one class of structures in which we might at first expect to realise the full theoretic gain even in the smallest examples. I refer to bridges for the sole purpose of carrying water-pipes, or channels for water supply or canal purposes. Further reflection will, however, show that this is not the case, for if arches be employed it will be necessary to have a distinct trough or tube, separate from but supported by the ribs, whereas if the girder principle be adopted the girders themselves may be made to assume the form of a trough or tube, thus dispensing with any separate structure to contain the water; and in this way the balance will be turned against the arch in the question of economy of material.

Let us now endeavour briefly to analyse the stresses endured by the material of an arched rib under varying conditions of temperature, load, &c.

We will first assume that the arch as originally designed is of a form adapted to the dead or unvarying load to be borne, which form in the usual case of a uniformly distributed load is a parabola having its axis vertical; and it may further be remarked that a circular curve will usually be found not to deviate in any important degree from the parabola, and is, from a practical point of view, decidedly preferable. Let us also assume that the rib is hinged at the crown as well as the springing. Let W represent the total weight of the structure, which may usually be taken as uniformly distributed over the whole length of the rib, b the span and h the rise of the arch; then the compression of the rib will be $\frac{Wl}{8h}$ at the crown, and at every other point $\frac{Wl \sec. \theta}{8h}$, when θ is the angle made by a tangent to the rib at the point in question with a horizontal line, and this compression will be uniformly distributed over the whole cross-section of the rib in every case. In other words, there will be no approach to a cross-bending action on any part of the rib, even though the temperature should vary or the abutments yield slightly to the thrust of the arch. If an additional load of W, uniformly distributed, be placed upon the bridge, these compressions will become $\frac{(W+W')\,l}{8h}$ and $\frac{(W+W')\,l \sec. \theta}{8h}$ respectively, and the perfect freedom from cross-bending before mentioned will still be maintained. If, however, the live load, instead of being uniformly distributed over the whole span, cover a part of it only, a cross-bending action will come into play, which will attain its maximum when half the bridge is loaded, and which will be unimportant or severe according as the live load is small or large compared with the weight of the structure. The tendency of this cross-bending action will be to increase the radius of curvature in the loaded side of the arch, as in Fig. 6, and reduce it in the unloaded; and the compression endured by the material of the rib will no longer be uniformly distributed, but will be greatly increased on the upper side of the loaded and the under side of the unloaded half of the rib. Hence, bearing in mind that either half of the arch may be the loaded portion, it is evident—1st. That the amount of metal in the rib must be increased. 2nd. That the best section for the rib is like a girder section consisting of two massive flanges united by a comparatively slight web. 3rd. That the rib should be made as deep as practical considerations will allow. The formulæ to be employed in computing the actual stresses in this case are too complex to be introduced here; they do not, of course, contain any terms representing change of temperature.

Let us now consider the behaviour of a rib hinged at the springing but continuous at the crown. When a load is imposed the metal will be compressed longitudinally, the rib will shorten, its crown will sink, and its radius of curvation increase (see dotted lines in Fig. 7), and any yielding of the abutments will tend to augment this result. The alteration in the radius of curvature implies a cross-bending action tending to increase the compression on the upper part of the rib, and to diminish it on the lower part, and this action will be present no matter how accurately the original form of the arch may have been adapted to the load to be carried. Let us now suppose the temperature to diminish. The crown of the arch will fall still further, the cross-bending action will be intensified, and the increasing inequality in the distribution of stress will produce a corresponding diminution in the available strength of the structure; the colder it becomes the more liable the bridge is to give way, and when fracture does ensue it will commence by the crushing of the upper part of the rib. We will next assume the temperature to increase. The crown of the arch will rise, its radius of curvation will be reduced, and the cross-bending action and consequent inequality of stress will diminish and ultimately vanish, and the arch will be stronger—*i.e.*, it will be able safely to bear a greater load than before; and under these conditions the formulæ quoted in the preceding case will apply to this also. A further increase of temperature will cause a further rise of the crown, and a further reduction of the radius of curvation, involving a cross-bending action in an opposite direction to that originally present, and a consequent inequality of stress and diminution in the power of the structure to endure a load. Thus the bridge will be best able to bear its load at a certain calculable temperature somewhat higher than that at which it was first put together, and its strength will fall off as this temperature is departed from in either direction. Hence we draw the inference that it is desirable to complete the erection of such an arch at a comparatively low temperature, in order that it may attain its maximum strength at or near the mean temperature to which it will be exposed. The engineer of the great St. Louis Bridge over the Mississippi enveloped the arch ribs in a kind of gigantic poultice of ice, in order to effect the final junction at a temperature sufficiently low.

The effect of a live load extending over a portion of the span will be the same as in the preceding case, the maximum effect being produced when the bridge is half-loaded and half-unloaded; the extra stresses due to the partial distribution of the live load being, of course, cumulative upon those due to temperature.* The most appropriate section for the rib will, as before, be a girder section; but we cannot say, as in the preceding case, that the deeper the rib the better, for great depth in the rib, while it will reduce the extra stresses due to partial loading, will increase those due to temperature, and a compromise will have to be made avoiding each extreme.

Having thus briefly detailed the considerations to be borne in mind when designing an iron arch, I will conclude by supplying a few particulars relative to a structure of the kind referred to, erected some time since by my friend Mr. T. E. Rawlinson, C.E., and which is, as far as I am aware, the only wrought-iron arched bridge in this colony.

This bridge is situated at Heidelberg on the River Yarra, and consists of a central opening originally occupied by a laminated wooden arch of 100 feet clear span and 17 feet rise and two lateral openings of smaller size. About three years ago the laminated arches gave way through decay of the timber; and Mr. Rawlinson, to whom the work of reconstruction had been entrusted, requested me to determine by computation the stresses on the proposed structure.

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* This is not mathematically correct, but is practically so for arches of the proportions commonly adopted by engineers.

It was in this way that my attention was first directed to this subject, and it is in compliance with a request made by him that I bring the subject before you to-night.

Figs. 8 and 9 respectively show a half-elevation and half-cross-section of the bridge to a scale of eight feet to one inch. The span, as before stated, is 100 feet in the clear, and the rise of the soffit of the arch twelve feet. The section of each flange of each of the two arched ribs is about twenty-four square inches at the crown, and increases slightly to the springing; and the web varies from $\frac{1}{4}$ inch thick at the crown to $\frac{1}{2}$ inch at the springing. The arches are continuous at the crown, but are probably capable of a very slight hinge action at the springing. Assuming them to be hinged at the springing, the following results have been obtained by calculation :—

1. Maximum compression of the metal, bridge half-loaded with load of 84 lbs. per square foot, at a temperature 40° below that at which it was erected =7180 lbs. per square inch.

2. When the load extends over the whole span the cross-bending stress vanishes at a temperature of about 16° Fahrenheit above that at which it was erected.

3. With a load extending half-way across, as in Fig. 6, the minimum stress occurs at a temperature 13° Fahrenheit above that at which the bridge was erected.

4. Ordinary plate girders to carry the same load would have contained from 30 to 40 per cent. more material than the iron arches.

The spandrils and roadway are constructed of timber as shown, and possess no doubt some stiffness and power of resisting the effect of irregular loads. In the previous calculations, however, no account was taken of this fact, it being considered unwise to rely upon two such different materials as wood and iron acting to any considerable degree in concert. The arch was therefore made strong enough to endure all irregular stresses without assistance from the spandrils.

In Fig. 10 a detailed section of one arched rib is given, and a portion of the lateral bracing connecting the two ribs together at intervals is shown.